# Introduction to PROBABILITY & STATISTICS

# MENDENHALL

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# Introduction to Probability and Statistics



EDITION

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#### Introduction to Probability and Statistics, Fourteenth Edition Mendenhall/Beaver/Beaver

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# Preface

Every time you pick up a newspaper or a magazine, watch TV, or surf the Internet, you encounter statistics. Every time you fill out a questionnaire, register at an online website, or pass your grocery rewards card through an electronic scanner, your personal information becomes part of a database containing your personal statistical information. You cannot avoid the fact that in this information age, data collection and analysis are an integral part of our day-to-day activities. In order to be an educated consumer and citizen, you need to understand how statistics are used and misused in our daily lives.

# THE SECRET TO OUR SUCCESS

The first college course in introductory statistics that we ever took used *Introduction to Probability and Statistics* by William Mendenhall. Since that time, this text—currently in the fourteenth edition—has helped several generations of students understand what statistics is all about and how it can be used as a tool in their particular area of application. The secret to the success of *Introduction to Probability and Statistics* is its ability to blend the old with the new. With each revision we try to build on the strong points of previous editions, while always looking for new ways to motivate, encourage, and interest students using new technological tools.

# HALLMARK FEATURES OF THE FOURTEENTH EDITION

The fourteenth edition retains the traditional outline for the coverage of descriptive and inferential statistics. This revision maintains the straightforward presentation of the thirteenth edition. In this spirit, we have continued to simplify and clarify the language and to make the language and style more readable and "user friendly"—without sacrificing the statistical integrity of the presentation. Great effort has been taken to explain not only how to apply statistical procedures, but also to explain

- how to meaningfully describe real sets of data
- what the results of statistical tests mean in terms of their practical applications
- how to evaluate the validity of the assumptions behind statistical tests
- what to do when statistical assumptions have been violated

#### **Exercises**

In the tradition of all previous editions, the variety and number of real applications in the exercise sets is a major strength of this edition. We have revised the exercise sets to provide new and interesting real-world situations and real data sets, many of which are drawn from current periodicals and journals. The fourteenth edition contains over 1300 problems, many of which are new to this edition. A set of classic exercises compiled from previous editions is available on the website (http://www.cengage. com/statistics/ mendenhall). Exercises are graduated in level of difficulty; some, involving only basic techniques, can be solved by almost all students, while others, involving practical applications and interpretation of results, will challenge students to use more sophisticated statistical reasoning and understanding.

#### **Organization and Coverage**

We believe that Chapters 1 through 10—with the possible exception of Chapter 3 should be covered in the order presented. The remaining chapters can be covered in any order. The analysis of variance chapter precedes the regression chapter, so that the instructor can present the analysis of variance as part of a regression analysis. Thus, the most effective presentation would order these three chapters as well.

Chapters 1–3 present descriptive data analysis for both one and two variables, using both *MINITAB* and Microsoft Excel<sup>®</sup> graphics. Chapter 4 includes a full presentation of probability and probability distributions. Three optional sections—Counting Rules, the Total Law of Probability, and Bayes' Rule—are placed into the general flow of text, and instructors will have the option of complete or partial coverage. The sections that present event relations, independence, conditional probability, and the Multiplication Rule have been rewritten in an attempt to clarify concepts that often are difficult for students to grasp. As in the thirteenth edition, the chapters on analysis of variance and linear regression include both calculational formulas and computer printouts in the basic text presentation. These chapters can be used with equal ease by instructors who wish to use the "hands-on" computational approach to linear regression and ANOVA and by those who choose to focus on the interpretation of computer-generated statistical printouts.

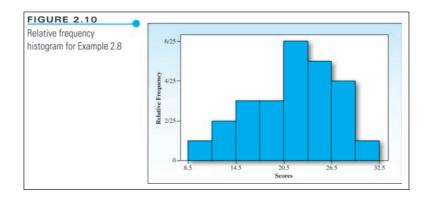
One important feature in the hypothesis testing chapters involves the emphasis on p-values and their use in judging statistical significance. With the advent of computergenerated p-values, these probabilities have become essential components in reporting the results of a statistical analysis. As such, the observed value of the test statistic and its p-value are presented together at the outset of our discussion of statistical hypothesis testing as equivalent tools for decision-making. Statistical significance is defined in terms of preassigned values of  $\alpha$ , and the p-value approach is presented as an alternative to the *critical value approach* for testing a statistical hypothesis. Examples are presented using both the p-value and *critical value* approaches to hypothesis testing. Discussion of the practical interpretation of statistical results, along with the difference between statistical significance and practical significance, is emphasized in the practical examples in the text.

#### **Special Features of the Fourteenth Edition**

NEED TO KNOW. . .: A special feature of this edition are highlighted sections called "NEED TO KNOW. . ." and identified by this icon. <a>[?]</a> NEED TO KNOW... These sections provide information consisting of definitions, procedures or step-by-step

hints on problem solving for specific questions such as "NEED TO KNOW... How to Construct a Relative Frequency Histogram?" or "NEED TO KNOW... How to Decide Which Test to Use?"

- Applets: Easy access to the Internet has made it possible for students to visualize statistical concepts using an interactive webtool called an applet. Applets written by Gary McClelland, author of *Seeing Statistics*<sup>TM</sup>, are found on the CourseMate CourseMate Website that accompanies the text. Following each applet, appropriate exercises are available that provide visual reinforcement of the concepts presented in the text. Applets allow the user to perform a statistical experiment, to interact with a statistical graph, to change its form, or to access an interactive "statistical table."
- Graphical and numerical data description includes both traditional and EDA methods, using computer graphics generated by *MINITAB 16* for Windows and MS Excel.



E	F	G	н	
Front Leg Room		Rear Leg Room		
Mean	41.333	Mean	28.944	
Standard Error	0.344	Standard Error	0.543	
Median	41	Median	29.5	
Mode	41	Mode	30	
Standard Deviation	1.031	Standard Deviation	1.629	
Sample Variance	1.063	Sample Variance	2.653	
Kurtosis	-0.745	Kurtosis	1.735	
Skewness	0.245	Skewness	-1.212	
Range	3	Range	5.5	
Minimum	40	Minimum	25.5	
Maximum	43	Maximum	31	
Sum	372	Sum	260.5	
Count	9	Count	9	

• All examples and exercises in the text contain printouts based on *MINITAB 16* and consistent with earlier versions of *MINITAB* or MS Excel. Printouts are provided for some exercises, while other exercises require the student to obtain solutions without using a computer.

304			Here is a list of the		
$\overline{\text{EX0147}}$ 44 presidents of the United States along with the number of regular vetoes used by each: <sup>5</sup>					
			•		
Washington	2	B. Harrison	19		
J. Adams	-	Cleveland	42 6		
Jefferson	0	McKinley	-		
Madison	5	T. Roosevelt	42		
Monroe	1	Taft	30		
J. Q. Adams	0 5	Wilson	33 5		
Jackson		Harding			
Van Buren	0	Coolidge	20		
W. H. Harrison	0	Hoover	21		
Tyler	6 2	F. D. Roosevelt			
Polk		Truman	180		
Taylor	0	Eisenhower	73		
Fillmore	0	Kennedy	12		
Pierce	9	L. Johnson	16		
Buchanan	4 2	Nixon	26		
Lincoln		Ford	48		
A. Johnson	21	Carter	13		
Grant	45	Reagan	39		
Hayes	12	G. H. W. Bush	29		
Garfield	0	Clinton	36		
Arthur	4	G. W. Bush	11		
Cleveland	304	Obama	1		
Source: The World	Almanac and	Book of Facts 2011			

Use an appropriate graph to describe the number of vetoes cast by the 44 presidents. Write a summary paragraph describing this set of data.

**1.48 Windy Cities** Are some cities more windy than others? Does Chicago deserve to be

(1950) 121.3 122.3 121.3 122.0 123.0 121.4 123.2 122.1 125.0 122.1 (1960) 122.2 124.0 120.2 121.4 120.0 121.1 122.0 120.3 122.1 121.4 (1970) 123.2 123.1 121.4 119.2<sup>+</sup> 124.0 122.0 121.3 122.1 122.2 121.1 (1980) 122.0 122.0 122.2 122.1 122.2 120.1 122.4 123.2 125.0 122.2 (1990) 122.0 123.0 123.0 122.2 123.3 121.1 121.0 122.4 122.2 123.2 (2000) 121.0 119.97 121.13 121.19 124.06 122.75 121.36 122.17 121.86 122.66 (2010) 124.4 <sup>†</sup>Record time set by Secretariat in 1973 Source: www.kentuckyderby.com a. Do you think there will be a trend in the winning times over the years? Draw a line chart to verify your answer. **b.** Describe the distribution of winning times using an

b. Describe the distribution of winning times using an appropriate graph. Comment on the shape of the distribution and look for any unusual observations.

**1.50** Gulf Oil Spill Cleanup On April 20, 2010, the United States experienced a major environmental disaster when a Deepwater Horizon drilling rig exploded in the Gulf of Mexico. The number of personnel and equipment used in the Gulf oil spill cleanup, beginning May 2, 2010 (Day 13) through June 9, 2010 (Day 51) is given in the following table.<sup>13</sup>

	Day 13	Day 26	Day 39	Day 51
Number of personnel (1000s)	3.0	17.5	20.0	24.0
Federal Gulf fishing areas closed	3%	8%	25%	32%
Booms laid (miles)	46	315	644	909
Dispersants used (1000 gallons)	156	500	870	1143



# The Role of Computers in the Fourteenth Edition—TECHNOLOGY TODAY

Computers are now a common tool for college students in all disciplines. Most students are accomplished users of word processors, spreadsheets, and databases, and they have no trouble navigating through software packages in the Windows environment. We believe, however, that advances in computer technology should not turn statistical analyses into a "black box." Rather, we choose to use the computational shortcuts and interactive visual tools that modern technology provides to give us more time to emphasize statistical reasoning as well as the understanding and interpretation of statistical results.

In this edition, students will be able to use computers for both standard statistical analyses and as a tool for reinforcing and visualizing statistical concepts. Both MS Excel and *MINITAB* 16 (consistent with earlier versions of *MINITAB*) are used exclusively as the computer packages for statistical analysis. However, we have chosen to isolate the instructions for generating computer output into individual sections called Technology Today at the end of each chapter. Each discussion uses numerical examples to guide the student through the MS Excel commands and option necessary for the procedures presented in that chapter, and then present the equivalent steps and commands needed to produce the same or similar results using *MINITAB*. We have included screen captures from both MS Excel and *MINITAB* 16, so that the student can actually work through these sections as "mini-labs."

If you do not need "hands-on" knowledge of *MINITAB* or MS Excel, or if you are using another software package, you may choose to skip these sections and simply use the printouts as guides for the basic understanding of computer printouts.

#### **TECHNOLOGY TODAY**

### Numerical Descriptive Measures in Excel

*MS Excel* provides most of the basic descriptive statistics presented in Chapter a single command on the **Data** tab. Other descriptive statistics can be calculate the **Function** command on the **Formulas** tab.



The following data are the front and rear leg rooms (in inches) for nine differer utility vehicles:<sup>14</sup>

Make & Model	Front Leg Room	Rear Leg Room
Acura MDX	41.0	28.5
Buick Enclave	41.5	30.0
Chevy TrailBlazer	40.0	25.5
Chevy Tahoe Hybrid V8 CVT	41.0	27.5
GMC Terrain 11 T 4-cvl	43 በ	31.0

#### Numerical Descriptive Measures in MINITAB

*MINITAB* provides most of the basic descriptive statistics presented in Chapter 2 using a single command in the drop-down menus.

The following data are the front and rear leg rooms (in inches) for nine different sports utility vehicles:<sup>14</sup>

Make and Model	Front Leg Room	Rear Leg Room
Acura MDX	41.0	28.5
Buick Enclave	41.5	30.0
Chevy TrailBlazer	40.0	25.5
Chevy Tahoe Hybrid V8 CVT	41.0	27.5
GMC Terrain 1LT 4-cyl	43.0	31.0
Honda CR-V	41.0	29.5
11 1.1 T	40 5	00 F

Any student who has Internet access can use the applets found on the CourseMate Website to visualize a variety of statistical concepts (access instructions for the CourseMate Website are listed on the Printed Access Card that is an optional bundle with this text). In addition, some of the applets can be used instead of computer software to perform simple statistical analyses. Exercises written specifically for use with these applets also appear on the CourseMate Website. Students can use the applets at home or in a computer lab. They can use them as they read through the text material, once they have finished reading the entire chapter, or as a tool for exam review. Instructors can use the applets as a tool in a lab setting, or use them for visual demonstrations during lectures. We believe that these applets will be a powerful tool that will increase student enthusiasm for, and understanding of, statistical concepts and procedures.

### **STUDY AIDS**

The many and varied exercises in the text provide the best learning tool for students embarking on a first course in statistics. The answers to all odd-numbered exercises are given in the back of the text, and a detailed solution appears in the *Student Solutions Manual*, which is available as a supplement for students. Each application exercise has

<b>a.</b> $P(x \le 3)$ <b>c.</b> $P(x = 3)$	d the following probabilities: <b>b.</b> $P(x > 3)$ <b>d.</b> $P(3 \le x \le 5)$ Poisson random variable with $\mu = 0.8$ .	<b>APPLICATIONS</b> <b>5.43 Airport Safety</b> The increased number of small commuter planes in major airports has heightened concern over air safety. An eastern airport has recorded a
Use Table 2 to fin <b>a.</b> $P(x = 0)$	d the following probabilities: <b>b.</b> $P(x \le 2)$	monthly average of five near misses on landings and takeoffs in the past 5 years.
<b>c.</b> $P(x > 2)$ <b>5.39</b> Let x be a F	<b>d.</b> $P(2 \le x \le 4)$ Poisson random variable with mean these probabilities:	<ul> <li>a. Find the probability that during a given month there are no near misses on landings and takeoffs at the airport.</li> <li>b. Find the probability that during a given month there.</li> </ul>
<b>a.</b> $P(x = 0)$ <b>c.</b> $P(x > 1)$	<b>b.</b> $P(x = 1)$ <b>d.</b> $P(x = 5)$	<ul><li>b. Find the probability that during a given month there are five near misses.</li><li>c. Find the probability that there are at least five near-</li></ul>

a title, making it easier for students and instructors to immediately identify both the context of the problem and the area of application.

Students should be encouraged to use the "NEED TO KNOW. . ." sections as they occur in the text. The placement of these sections is intended to answer questions as they would normally arise in discussions. In addition, there are numerous hints called "NEED A TIP?" that appear in the margins of the text. The tips are short and concise.

NEED A TIP	Is Tchebysheff's Theorem applicable? Yes, because it can be used for any set of dota. According to Tchebysheff's Theorem
Empirical Rule ⇔ mound-shaped data	<ul> <li>data. According to Tchebysheff's Theorem,</li> <li>at least 3/4 of the measurements will fall between 10.6 and 32.6.</li> </ul>
Tchebysheff ⇔ any shaped data	• at least 8/9 of the measurements will fall between 5.1 and 38.1.

Finally, sections called **Key Concepts and Formulas** appear in each chapter as a review in outline form of the material covered in that chapter.

<	ey Concepts and Formulas	68%, 95%, and 99.7% of the measurements are within one, two, and three standard deviations		
Ι.	Measures of the Center of a Data Distribution	of the mean, respectively.		
	1. Arithmetic mean (mean) or average	IV. Measures of Relative Standing		
	a. Population: $\mu$ b. Sample of <i>n</i> measurements: $\overline{x} = \frac{\sum x_i}{n}$	1. Sample <i>z</i> -score: $z = \frac{x - \overline{x}}{s}$		
	2. Median; <b>position</b> of the median = $.5(n + 1)$ 3. Mode	<ol> <li><i>p</i>th percentile; <i>p</i>% of the measurements are smaller, and (100 - <i>p</i>)% are larger.</li> </ol>		
	4. The median may be preferred to the mean if the	3. Lower quartile, $Q_1$ ; position of $Q_1 = .25 (n + 1)$		
п.	data are highly skewed. Measures of Variability	4. Upper quartile, $Q_3$ ; <b>position</b> of $Q_3 = .75 (n + 1)$		
	1. Range: $R = \text{largest} - \text{smallest}$	5. Interquartile range: $IQR = Q_3 - Q_1$		
	2. Variance a. Population of <i>N</i> measurements: $\sigma^{2} = \frac{\Sigma(x_{i} - \mu)^{2}}{N}$	V. The Five-Number Summary and Box Plots		
		1. The five-number summary:		
	N	Min $Q_1$ Median $Q_3$ Max		
	b. Sample of <i>n</i> measurements: $s^{2} = \frac{\Sigma(x_{i} - \overline{x})^{2}}{n - 1} = \frac{\Sigma x_{i}^{2} - \frac{(\Sigma x_{i})^{2}}{n}}{n - 1}$	One-fourth of the measurements in the data set lie between each of the four adjacent pairs of numbers.		
	n-1 $n-1$	2. Box plots are used for detecting outliers and		

The CourseMate Website, a password-protected resource that can be accessed with a Printed Access Card (optional bundle item), provides students with an array of study resources, including the complete set of Java applets, the TI Calculator Tech Guide that includes instructions for performing many of the techniques in the text using the popular TI 83/84/89 graphing calculator, an interactive eBook, online Quizzes, flashcards, and more. The data sets (saved in a variety of formats) can be found on the book's website (www.CengageBrain.com) as well as the CourseMate Website.

### **INSTRUCTOR RESOURCES**

The **Instructor's Website** (http://www.cengage.com/statistics/mendenhall), available to adopters of the fourteenth edition, provides a variety of teaching aids, including

- All the material from the CourseMate website including exercises using the Large Data Sets, which is accompanied by three large data sets that can be used throughout the course. A file named "Fortune" contains the revenues (in millions) for the *Fortune* 500 largest U.S. industrial corporations in a recent year; a file named "Batting" contains the batting averages for the National and American baseball league batting champions from 1976 to 2010; and a file named "Blood Pressure" contains the age and diastolic and systolic blood pressures for 965 men and 945 women compiled by the National Institutes of Health.
- Classic exercises with data sets and solutions
- PowerPoint lecture slides
- Applets by Gary McClelland (the complete set of Java applets used for the MyApps exercises on the website)
- TI Calculator Tech Guide, which includes instructions for performing many of the techniques in the text using the TI-83/84/89 graphing calculators.

Also available for instructors:

#### Aplia

Aplia is a web-based learning solution that increases student effort and engagement. It helps make statistics relevant and engaging to students by connecting real-world examples to course concepts. When combined with the textual material of *Introduction to Probability and Statistics* (IPS) 14,

- Students receive immediate, detailed explanations for every answer.
- Math and graphing tutorials help students ovecome deficiencies in these crucial areas.
- Grades are automatically recorded in the instructor's Aplia gradebook.

#### **Solution Builder**

This online instructor database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class. Sign up for access at www.cengage.com/solutionbuilder.

#### **PowerLecture**<sup>TM</sup>

PowerLecture with ExamView<sup>®</sup> for *Introduction to Probability and Statistics* contains the Instructor's Solutions Manual, PowerPoint lectures, ExamView Computerized Testing, Classic Exercises, and TI-83/84/89 calculator Tech Guide which includes instructions for performing many of the techniques in the text using the TI-83/84/89 graphing calculators.

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Robert J. Beaver Barbara M. Beaver

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# Introduction What is Statistics?

What is statistics? Have you ever met a statistician? Do you know what a statistician does? Perhaps you are thinking of the person who sits in the broadcast booth at the Rose Bowl, recording the number of pass completions, yards rushing, or interceptions thrown on New Year's Day. Or perhaps the mere mention of the word *statistics* sends a shiver of fear through you. You may think you know nothing about statistics; however, it is almost inevitable that you encounter statistics in one form or another every time you pick up a daily newspaper. Here are some examples concerning the California 2010 elections:

- Rowdy crowd jeers Whitman. GOP candidate criticizes unions; earlier stop draws friendlier audience.
   GLENDALE ... Whitman, a billionaire, has spent \$142 million from her personal fortune in the race so far.
   A Field Poll released Thursday showed her trailing Jerry Brown 49 percent to 39 percent among likely voters.<sup>1</sup>
- Fiorina calls herself similar to Feinstein, who supports Boxer.

**MENLO PARK**—Republican Carly Fiorina said Friday she would be a like-minded colleague of Democratic Sen. Dianne Feinstein if she unseats Barbara Boxer next week, drawing sharp responses from both Democratic senators. ... Fiorina, the former CEO of Hewlett-Packard Co., disputed a Field Poll released Friday showing Boxer leading her among likely voters, 49 percent to 41 percent.<sup>2</sup>

• Race for attorney general tight. Field Poll: Nearly a quarter of those surveyed are undecided. Newsom holds a slim lead over Maldonado for lieutenant governor.



© Mark Karrass/CORBIS

**SACRAMENTO**—Tuesday's election for attorney general is a tossup, with Democrat Kamala Harris and Republican Steve Cooley virtually tied as Harris gains ground in voter-rich Los Angeles County and among women according to the latest Field Poll.

... Today's poll shows Cooley with 39 percent and Harris with 38 percent among likely voters. Almost a quarter of likely voters remain undecided.

... Newsom, the mayor of San Francisco, leads Maldonado, who was appointed lieutenant governor this year, 42 percent to 37 percent. A fifth of voters are undecided.

Today's poll was conducted for *The Press-Enterprise* and other California media subscribers. It was conducted October 14 through October 26 and included 1092 voters. It has a margin of error of plus or minus 3.2 percent.<sup>3</sup>

-The Press-Enterprise, Riverside, CA

Articles similar to these are commonplace in our newspapers and magazines, and in the period just prior to a presidential or congressional election, a new poll is reported almost every day. The language of these articles are very familiar to us; however, they leave the inquisitive reader with some unanswered questions. How were the people in the poll selected? Will these people give the same response tomorrow? Will they give the same response on election day? Will they even vote? Are these people representative of all those who will vote on election day? It is the job of a statistician to ask these questions and to find answers for them in the language of the poll.

#### Most Believe "Cover-Up" of JFK Assassination Facts

A majority of the public believes the assassination of President John F. Kennedy was part of a larger conspiracy, not the act of one individual. In addition, most Americans think there was a cover-up of facts about the 1963 shooting. Almost 50 years after JFK's assassination, a FOX news poll shows many Americans disagree with the government's conclusions about the killing. The **Warren Commission** found that **Lee Harvey Oswald** acted alone when he shot Kennedy, but 66 percent of the public today think the assassination was "part of a larger conspiracy" while only 25 percent think it was the "act of one individual."

"For older Americans, the Kennedy assassination was a traumatic experience that began a loss of confidence in government," commented Opinion Dynamics President John Gorman. "Younger people have grown up with movies and documentaries that have pretty much pushed the 'conspiracy' line. Therefore, it isn't surprising there is a fairly solid national consensus that we still don't know the truth."

	We Know All the Facts (%)	There Was a Cover-Up	(Not Sure)
All	14	74	12
Democrats	11	81	8
Republicans	18	69	13
Independents	12	71	17

(The poll asked): "Do you think that we know all the facts about the assassination of President John F. Kennedy or do you think there was a cover-up?"

-www.foxnews.com4

When you see an article like this one in a magazine, do you simply read the title and the first paragraph, or do you read further and try to understand the meaning of the numbers? How did the authors get these numbers? Did they really interview every American with each political affiliation? It is the job of the statistician to interpret the language of this study.

#### Hot News: 98.6 Not Normal

After believing for more than a century that 98.6 was the normal body temperature for humans, researchers now say normal is not normal anymore.

For some people at some hours of the day, 99.9 degrees could be fine. And readings as low as 96 turn out to be highly human.

The 98.6 standard was derived by a German doctor in 1868. Some physicians have always been suspicious of the good doctor's research. His claim: 1 million readings—in an epoch without computers.

So Mackowiak & Co. took temperature readings from 148 healthy people over a three-day period and found that the mean temperature was 98.2 degrees. Only 8 percent of the readings were 98.6.

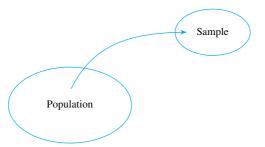
#### *—The Press-Enterprise*<sup>5</sup>

What questions come to your mind when you read this article? How did the researcher select the 148 people, and how can we be sure that the results based on these 148 people are accurate when applied to the general population? How did the researcher arrive at the normal "high" and "low" temperatures given in the article? How did the German doctor record 1 million temperatures in 1868? Again, we encounter a statistical problem with an application to everyday life.

Statistics is a branch of mathematics that has applications in almost every facet of our daily life. It is a new and unfamiliar language for most people, however, and, like any new language, statistics can seem overwhelming at first glance. But once the language of statistics is learned and understood, it provides a powerful tool for data analysis in many different fields of application.

### THE POPULATION AND THE SAMPLE

In the language of statistics, one of the most basic concepts is **sampling.** In most statistical problems, a specified number of measurements or data—a **sample**—is drawn from a much larger body of measurements, called the **population.** 



For the body-temperature experiment, the sample is the set of body-temperature measurements for the 148 healthy people chosen by the experimenter. We hope that the sample is representative of a much larger body of measurements—the population—the body temperatures of all healthy people in the world!

Which is of primary interest, the sample or the population? In most cases, we are interested primarily in the population, but the population may be difficult or impossible to enumerate. Imagine trying to record the body temperature of every healthy person on earth or the presidential preference of every registered voter in the United States! Instead, we try to describe or predict the behavior of the population on the basis of information obtained from a representative sample from that population.

The words *sample* and *population* have two meanings for most people. For example, you read in the newspapers that a Gallup poll conducted in the United States was based on a sample of 1823 people. Presumably, each person interviewed is asked a particular question, and that person's response represents a single measurement in the sample. Is the sample the set of 1823 people, or is it the 1823 responses that they give?

In statistics, we distinguish between the set of objects on which the measurements are taken and the measurements themselves. To experimenters, the objects on which measurements are taken are called **experimental units**. The sample survey statistician calls them **elements of the sample**.

# DESCRIPTIVE AND INFERENTIAL STATISTICS

When first presented with a set of measurements—whether a sample or a population you need to find a way to organize and summarize it. The branch of statistics that presents techniques for describing sets of measurements is called **descriptive statistics.** You have seen descriptive statistics in many forms: bar charts, pie charts, and line charts presented by a political candidate; numerical tables in the newspaper; or the average rainfall amounts reported by the local television weather forecaster. Computer-generated graphics and numerical summaries are commonplace in our everyday communication.

**Definition Descriptive statistics** consists of procedures used to summarize and describe the important characteristics of a set of measurements.

If the set of measurements is the entire population, you need only to draw conclusions based on the descriptive statistics. However, it might be too expensive or too time consuming to enumerate the entire population. Perhaps enumerating the population would destroy it, as in the case of "time to failure" testing. For these or other reasons, you may have only a sample from the population. By looking at the sample, you want to answer questions about the population as a whole. The branch of statistics that deals with this problem is called **inferential statistics**.

**Definition** Inferential statistics consists of procedures used to make inferences about population characteristics from information contained in a sample drawn from this population.

The **objective of inferential statistics** is to make inferences (that is, draw conclusions, make predictions, make decisions) about the characteristics of a population from information contained in a sample.



# ACHIEVING THE OBJECTIVE OF INFERENTIAL STATISTICS: THE NECESSARY STEPS

How can you make inferences about a population using information contained in a sample? The task becomes simpler if you organize the problem into a series of logical steps.

1. **Specify the questions to be answered and identify the population of interest.** In the California election poll, the objective is to determine who will get the most votes on election day. Hence, the population of interest is the set of all votes in the California election. When you select a sample, it is important that the sample be representative of *this* population, not the population of voter preferences on October 30 or on some other day prior to the election.

- 2. Decide how to select the sample. This is called the *design of the experiment* or the *sampling procedure*. Is the sample representative of the population of interest? For example, if a sample of registered voters is selected from the city of San Francisco, will this sample be representative of all voters in California? Will it be the same as a sample of "likely voters"—those who are likely to actually vote in the election? Is the sample large enough to answer the questions posed in step 1 without wasting time and money on additional information? A good sampling design will answer the questions posed with minimal cost to the experimenter.
- 3. Select the sample and analyze the sample information. No matter how much information the sample contains, you must use an appropriate method of analysis to extract it. Many of these methods, which depend on the sampling procedure in step 2, are explained in the text.
- 4. Use the information from step 3 to make an inference about the population. Many different procedures can be used to make this inference, and some are better than others. For example, 10 different methods might be available to estimate human response to an experimental drug, but one procedure might be more accurate than others. You should use the best inference-making procedure available (many of these are explained in the text).
- 5. Determine the reliability of the inference. Since you are using only a fraction of the population in drawing the conclusions described in step 4, you might be wrong! How can this be? If an agency conducts a statistical survey for you and estimates that your company's product will gain 34% of the market this year, how much confidence can you place in this estimate? Is this estimate accurate to within 1, 5, or 20 percentage points? Is it reliable enough to be used in setting production goals? Every statistical inference should include a measure of reliability that tells you how much confidence you have in the inference.

Now that you have learned a few basic terms and concepts, we again pose the question asked at the beginning of this discussion: Do you know what a statistician does? The statistician's job is to implement all of the preceding steps.



# **KEYS FOR SUCCESSFUL LEARNING**

As you begin to study statistics, you wiil find that there are many new terms and concepts to be mastered. Since statistics is an applied branch of mathematics, many of these basic concepts are mathematical—developed and based on results from calculus or higher mathematics. However, you do not have to be able to derive results in order to apply them in a logical way. In this text, we use numerical examples and commonsense arguments to explain statistical concepts, rather than more complicated mathematical arguments.

In recent years, computers have become readily available to many students and provide them with an invaluable tool. In the study of statistics, even the beginning student can use packaged programs to perform statistical analyses with a high degree of speed and accuracy. Some of the more common statistical packages available at computer facilities are *MINITAB*<sup>TM</sup>, SAS (Statistical Analysis System), and SPSS